## \&structural Basics

## STRUCTURAL DESIGN CHEATSHEET

A summary of the most important formulas, equations \& diagrams

$$
M=\frac{q l^{2}}{8}
$$



## DISCLAIMER

This Structural Design Cheatsheet covers the engineering formulas and equations I use regularly as a structural engineer. It's a "short" summary of many of the blog posts we published on structuralbasics.com.

While this document covers many important formulas, be aware that it doesn't cover every formula. And as for any structural engineering book or formula sheet, some explanations and formulas are simplified. For example, the wind load is only shown from one direction, or the steel Eurocode provides also formulas for bending and shear, normal force and bending, etc.

Structural engineering is simply too complex to cover every design situation.

Use the content of the document as a guide, but question everything critically.

This document is supplied 'as-is,' without any express or implied warranty regarding the accuracy or completeness of the information it contains. The user assumes all risks associated with its use. The author will not be held responsible for any damage or harm to any person or entity that may come from utilizing this document.

With that out of the way, I hope this document helps you get a good overview of structural design.

## Simply supported beam - Line load



Reation forces:
$R_{a}=R_{b}=q \frac{l}{2}$
Max. shear forces:
$V_{a}=V_{b}=q \frac{l}{2}$
Max. bending moment:
$M_{\max }=q \frac{l^{2}}{8}$

## Simply supported beam - Point load



Reation forces:
$R_{a}=Q \frac{b}{l}, R_{b}=Q \frac{a}{l}$
Shear forces:
$V_{a}=Q \frac{b}{l}, V_{b}=Q \frac{a}{l}$

Max. bending moment:
$M_{\text {max }}=Q \frac{a \cdot b}{l}$
For more moment and shear force formulas of the simply supported beam click here.

## Cantilever beam - Line load

q


Max. bending moment:
$M_{\text {max }}=-\frac{1}{2} \cdot q \cdot l^{2}$

## STATICS

## Cantilever beam - Point load



For more moment and shear force formulas of the cantilever beam click here.

## 2 span continuous beam - Line load on 2 spans



Reation forces:
$R_{a}=R_{c}=\frac{3}{8} q \cdot l, R_{b}=\frac{5}{4} q \cdot l$
Shear forces:
$V_{a}=V_{c}=\frac{3}{8} q \cdot l, V_{b}=\frac{5}{8} q \cdot l$

Bending moments:
$M_{\text {max }}=\frac{9}{128} \cdot q \cdot l^{2}, M_{b}=-\frac{1}{8} \cdot q \cdot l^{2}$

## 2 span continuous beam - Line load on 1 span



Reation forces:
$R_{a}=\frac{7}{16} q \cdot l, R_{b}=\frac{5}{4} q \cdot l$,
$R_{c}=-\frac{1}{16} q \cdot l$
Shear forces:
$V_{a}=R_{a}, V_{b}=-\frac{9}{16} q \cdot l, V_{c}=-R_{c}$
Max. bending moment:
$M_{\max }=\frac{49}{512} \cdot q \cdot l^{2}, M_{b}=-\frac{1}{16} \cdot q \cdot l^{2}$
For more moment and shear force formulas of the 2 span continuous beam click here.

## STATICS

## 3 span continuous beam - Line load on 3 spans



Reation forces:
$R_{a}=R_{d}=0.4 \cdot q \cdot l$,
$R_{b}=R_{c}=1.1 \cdot q \cdot l$
Shear forces:
$V_{a}=V_{d}= \pm 0.4 \cdot q \cdot l$,
$V_{b(-)}=-V_{c(+)}=-0.6 \cdot q \cdot l$,
$V_{b(+)}=-V_{c(-)}=0.5 \cdot q \cdot l$
Bending moment:
$M_{\max }=0.08 \cdot q \cdot l^{2}, M_{b}=-0.1 \cdot q \cdot l^{2}$

## 3 span continuous beam - Line load on 2 spans



Reation forces:
$R_{a}=0.383 \cdot q \cdot l, R_{b}=1.2 \cdot q \cdot l$,
$R_{c}=0.45 \cdot q \cdot l, R_{d}=-0.033 \cdot q \cdot l$
Shear forces:
$V_{a}=0.383 \cdot q \cdot l, V_{d}=0.033 \cdot q \cdot l$
$V_{b(-)}=-0.617 q l, V_{b(+)}=-0.583 q l$
$V_{c(-)}=-0.417 q l, V_{c(+)}=0.033 q l$
Bending moments:
$M_{\text {max }}=0.074 q l^{2}, M_{b}=-0.117 q l^{2}$,
$M_{b c}=0.053 q l^{2}, M_{c}=-0.033 q l^{2}$
3 span continuous beam - Line load on 1 span


Reation forces:
$R_{a}=0.433 \cdot q \cdot l, R_{b}=0.65 \cdot q \cdot l$,
$R_{C}=-0.1 \cdot q \cdot l, R_{d}=0.017 \cdot q \cdot l$

Bending moments:
$M_{\text {max }}=0.094 q l^{2}, M_{b}=-0.067 q l^{2}$,
$M_{c}=0.017 q l^{2}$

For more moment and shear force formulas of the 3 span continuous beam click here.

## GEOMETRY

## Centroid

The centroid is a point of a cross-section that represents the center of mass. It's the point at which the entire area of the section can be assumed to be concentrated.


$$
z=\frac{\sum_{i=1}^{n} A_{i} \cdot z_{i}}{A}
$$

With,
n
$\mathrm{A}_{\mathrm{i}}$
$z_{i}$

A
number of "parts" of the cross-section ( $\mathrm{n}=2$ in the shape above) area of the part i
distance of the centroid of part i from the point of origin (top fibre in the shape above)
area of the section

## Moment of inertia

The moment of inertia is a measure of an object's resistance to changes in rotational motion. It is used to calculate the bending stresses that a structural element will experience when subjected to a load.


$$
I=\sum_{i=1}^{n} I_{0 . i}+A_{i} \cdot z_{i}^{2}
$$

With,
number of "parts" of the cross-section ( $n=2$ in the shape above) moment of inertia of part i
area of the part i
distance of the centroid of part i from the centroid $c$

## GEOMETRY

## Section modulus



Section modulus of top fibre:
$W=\frac{I}{Z_{1}}$
Section modulus of bottom fibre:
$W=\frac{I}{Z_{2}}$

With,

I
moment of inertia around strong axis
z distance of fibre from centroid

## Moment of inertia formulas

Rectangular section


$$
\begin{aligned}
& I_{y}=\frac{w h^{3}}{12} \\
& I_{z}=\frac{h w^{3}}{12}
\end{aligned}
$$

Hollow circular section


$$
\begin{aligned}
& I_{y}=\frac{\left(D^{4}-d^{4}\right) \cdot \pi}{64} \\
& I_{z}=\frac{\left(D^{4}-d^{4}\right) \cdot \pi}{64}
\end{aligned}
$$

I/H section


Rectangular hollow section


$$
I_{y}=\frac{B H^{3}-b h^{3}}{12}
$$

$$
I_{z}=\frac{H B^{3}-h b^{3}}{12}
$$

For more moment of inertia formulas of other sections click here.

## LOADS

## Overview of loads used in structural design

The "common" characteristic loads that are used in the structural design of buildings are:

- Dead load
- Live load
- Horizontal wind load on walls
- Wind loads on roofs
- Snow load
- Soil pressure
- Seismic load


## Dead load

The dead load represents permanent loads, such as the self-weight of structural and non-structural building materials. The self-weight of a concrete slab, a timber truss roof and windows are examples of the dead load. The weight is calculated and then applied to the structural member that carries it.

Area dead load:

$$
g_{k}=\text { density of element } \cdot \text { thickness }\left[\frac{k N}{m}\right]
$$

Click here for the in-depth article.

## Live load

The live load represents variable loads such as weight of people, furniture, cars, office equipment, etc. that can change over time. It's an approximation for structural engineers to estimate the additional weight (excluding self-weight) that can act on structures due to different room categories.

You'll find the values of the live load for different room classes in EN 1991-1-1 Table 6.2 and the National Annex of your country.

Click here for the in-depth article.

## Horizontal wind loads on walls

The horizontal wind load on buildings are split up in 4 or 5 areas (A, B, C, D, E) with different load values.


Wind loads on walls:

$$
w_{k}=q_{p} \cdot c_{p e}
$$

With,
$q_{p} \quad$ peak velocity pressure
$\mathrm{C}_{\mathrm{pe}}$
pressure coefficient for each area according to EN 1991-
1-4 Table 7.1
Click here for step-by-step guide for the peak velocity pressure.
And here for the full calculation guide for horizontal wind loads.

## LOADS

## Wind loads on pitched roofs

The wind loads on pitched roofs are split up in 4 or 5 areas ( $\mathrm{F}, \mathrm{G}, \mathrm{H}, \mathrm{I},(\mathrm{J})$ ) with different load values.


Wind loads on pitched roofs:

$$
w_{k}=q_{p} \cdot c_{p e}
$$

With,
peak velocity pressure
pressure coefficient for each area according to EN 1991-1-4 Table 7.4a.

Click here for the calculation guide for wind loads on pitched roofs.

## LOADS

## Wind loads on flat roofs

The wind loads on flat roofs are split up in 4 areas ( $\mathrm{F}, \mathrm{G}, \mathrm{H}, \mathrm{I}$ ) with different load values.


Wind loads on flat roofs:

$$
w_{k}=q_{p} \cdot c_{p e}
$$

With,
$q_{p}$
$\mathrm{C}_{\mathrm{pe}}$
peak velocity pressure
pressure coefficient for each area according to EN 1991-1-4 Table 7.2.

Click here for the calculation guide for wind loads on flat roofs.

## Snow loads on flat and pitched roofs

The snow load on pitched and flat roofs is calculated as (EN 1991-1-3 (5.1)):

$$
s=\mu_{i} \cdot C_{e} \cdot C_{t} \cdot s_{k}
$$

With,

| $\mu_{i}$ | snow load shape coefficient |
| :--- | :--- |
| $C_{e}$ | exposure coefficient |
| $C_{t}$ | thermal coefficient |
| $S_{k}$ | characteristic snow load value on ground |

Click here for a calculation guide for snow loads on pitched roofs. And here for flat roofs.

## LOAD COMBINATIONS

## Ultimate limit state (ULS)

$$
\sum_{j \geq 1} \gamma_{G . j} \cdot G_{k . j}+\gamma_{Q .1} \cdot Q_{k .1}+\sum_{i>1} \gamma_{Q . i} \cdot \psi_{0 . i} \cdot Q_{k . i}
$$

With,
$\gamma_{G}$
$G_{k}$
$\mathrm{Y}_{\mathrm{a}}$ $Q_{k}$
$\Psi_{0}$
partial factor for permanent actions
characteristic value of permanent action (e.g. dead load) partial factor for variable actions characteristic value of variable action (e.g. live, snow and wind load)
factor for combination value of a variable action

## Serviceability limit state (SLS)

Characteristic combination:

$$
\sum_{j \geq 1} G_{k . j}+Q_{k .1}+\sum_{i>1} \psi_{0 . i} \cdot Q_{k . i}
$$

Frequent combination:

$$
\sum_{j \geq 1} G_{k . j}+\psi_{1.1} \cdot Q_{k .1}+\sum_{i>1} \psi_{2 . i} \cdot Q_{k . i}
$$

Quasi-permanent combination:

$$
\sum_{j \geq 1} G_{k . j}+\sum_{i \geq 1} \psi_{2 . i} \cdot Q_{k . i}
$$

With, factor for quasi-permanent value of a variable action

## LOAD COMBINATIONS

## Accidental limit state (ALS)

$$
\sum_{j \geq 1} G_{k . j}+A_{d}+\left(\psi_{1.1} \text { or } \psi_{2.1}\right) \cdot Q_{k .1}+\sum_{i>1} \psi_{2 . i} \cdot Q_{k . i}
$$

Seismic combination:

$$
\sum_{j \geq 1} G_{k . j}+A_{E d}+\sum_{i \geq 1} \psi_{2 . i} \cdot Q_{k . i}
$$

With,

| $A_{d}$ | design value of an accidental action |
| :--- | :--- |
| $A_{E d}$ | design value of seismic action |

Full in-depth article about load combinations: here

## Bending verification

Design bending stresses:

$$
\begin{aligned}
& \sigma_{m \cdot y \cdot d}=\frac{M_{y \cdot d}}{I_{y}} \cdot \frac{h}{2} \\
& \sigma_{m \cdot z \cdot d}=\frac{M_{z . d}}{I_{z}} \cdot \frac{w}{2}
\end{aligned}
$$

With,

| $M_{d}$ | design bending moment around $y / z$-axis |
| :--- | :--- |
| $I$ | moment of inertia around $y / z$-axis |
| $h$ | cross-sectional height |
| $w$ | cross-sectional width |

Design bending strengths (EN 1995-1-1 (2.17)):

$$
\begin{aligned}
f_{m \cdot y \cdot d} & =k_{m o d} \frac{f_{m \cdot y \cdot k}}{\gamma_{m}} \\
f_{m . z . d} & =k_{m o d} \frac{f_{m \cdot z \cdot k}}{\gamma_{m}}
\end{aligned}
$$

With,
modification factor (EN 1995-1-1 Table 3.1)
characteristic bending strength of timber material partial factor (EN 1995-1-1 Table 2.3)

Bending verification (EN 1995-1-1 (6.11) + (6.12))

$$
\begin{aligned}
& k_{m}=0.7 / 1.0 \\
& \frac{\sigma_{m . y . d}}{f_{m \cdot y \cdot d}}+k_{m} \cdot \frac{\sigma_{m . z . d}}{f_{m . z . d}} \leq 1 \\
& k_{m} \cdot \frac{\sigma_{m \cdot y \cdot d}}{f_{m . y . d}}+\frac{\sigma_{m . z . d}}{f_{m . z . d}} \leq 1
\end{aligned}
$$

## Shear verification

Design shear stress:

$$
\tau_{v . d}=\frac{3}{2} \cdot \frac{V_{d}}{w \cdot h}
$$

With,
$V_{d} \quad$ design shear force
h cross-sectional height
w cross-sectional width

Design shear strengths (EN 1995-1-1 (2.17)):

$$
f_{v . d}=k_{\bmod } \frac{f_{v . k}}{\gamma_{m}}
$$

With,
$\mathrm{k}_{\text {mod }}$
$\mathrm{f}_{\mathrm{v} . \mathrm{k}}$
$\gamma_{m}$
modification factor (EN 1995-1-1 Table 3.1) characteristic shear strength of timber material partial factor (EN 1995-1-1 Table 2.3)

Shear verification (EN 1995-1-1 (6.13))

$$
\frac{\tau_{v . d}}{f_{v . d}} \leq 1
$$

## Compression verification (columns)

Design compression stress:

$$
\sigma_{c .0 . d}=\frac{N_{d}}{w \cdot h}
$$

With,
$\mathrm{N}_{\mathrm{d}} \quad$ design normal force
h cross-sectional height
w cross-sectional width

Design compression strength - parallel to grain (EN 1995-1-1 (2.17)):

$$
f_{c .0 . d}=k_{m o d} \frac{f_{c .0 . k}}{\gamma_{m}}
$$

With,
$k_{\text {mod }}$
$\mathrm{f}_{\mathrm{c} .0 . \mathrm{k}}$
$\gamma_{m}$
modification factor (EN 1995-1-1 Table 3.1)
characteristic compression strength parallel to grain partial factor (EN 1995-1-1 Table 2.3)

Compression verification (EN 1995-1-1 (6.2))

$$
\frac{\sigma_{c .0 . d}}{f_{c .0 . d}} \leq 1
$$

## TIMBER DESIGN

## Buckling verification (columns)

Buckling length:
$l$

Radius of inertia:

Slenderness ratio:
$i=\sqrt{\frac{I}{w \cdot h}}$
$\lambda=\frac{l}{i}$
Relative slenderness ratio (EN $\quad \lambda_{\text {rel.y }}=\frac{\lambda}{\pi} \cdot \sqrt{\frac{f_{c .0 . k}}{E_{0 . g .05}}}$
1995-1-1 (6.21)):
$\beta_{c}$ factor (EN 1995-1-1 (6.29)) $\quad \beta_{c}$

Instability factor (EN 1995-1-1 $k=0.5 \cdot\left(1+\beta_{c} \cdot\left(\lambda_{r e l . y}-0.3\right)+\lambda_{r e l . y}^{2}\right)$ (6.27))

Buckling reduction factor coefficient (EN 1995-1-1 (6.25))

$$
k_{c}=\frac{1}{k+\sqrt{k^{2}-\lambda_{r e l . y}^{2}}}
$$

Utilization check one axial bending (EN 1995-1-1 (6.23))

$$
\frac{\sigma_{c .0 . d}}{k_{c} \cdot f_{c .0 . d}}+\frac{\sigma_{m . d}}{f_{m . d}} \leq 1
$$

With,

| $\mathrm{f}_{\text {c.0.k }}$ | characteristic compression strength parallel to grain |
| :--- | :--- |
| $\mathrm{E}_{\text {o.g.05 }}$ | E-modulus parallel to grain, $5 \%$ fractile |
| I | moment of inertia |
| h | cross-sectional height |
| w | cross-sectional width |

## STEEL DESIGN

## Material properties

E-modulus:

$$
E=0.21 \cdot 10^{6} \mathrm{MPa}
$$

Density:

$$
\rho=7850 \frac{\mathrm{~kg}}{\mathrm{~m}^{3}}
$$

Poissons ratio:

$$
v=0.3
$$

Shear modulus:

$$
G=\frac{E}{2 \cdot(1+v)}
$$

Yield and ultimate strength: $\quad f_{y}, f_{u}$

Check out the tables on Eurocodeapplied.com (link)

Strength properties of bolts (EN 1993-1-8 Table 3.1)

|  | Bolt class |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Property | 4.6 | 4.8 | 5.6 | 5.8 | 6.8 | 8.8 | 10.9 |
| Yield strength $\mathrm{f}_{\mathrm{yb}}(\mathrm{MPa})$ | 240 | 320 | 300 | 400 | 480 | 640 | 900 |
| Ultimate tensile <br> strength $\mathrm{f}_{\mathrm{ub}}(\mathrm{MPa})$ | 400 | 400 | 500 | 500 | 600 | 800 | 1000 |

Ultimate tensile strength of welds:

Correlation factor
$\beta_{0}$
(dependent on steel strength
(EN 1993-1-8 Table 4.1)

## STEEL DESIGN

## Geometric properties

Cross-sectional height: $h$

Cross-sectional width:
w

Web thickness:
$t_{w}$

Flange thickness:
$t_{f}$

Root radius:
$r$

You can find these and many more parameters on Eurocodeapplied.com (link)

Bolt nominal diameter:
$d$

Bolt Hole diameter:
$d_{0}$

Stress area (threaded part of $A_{S}$ bolt):

You can find these and many more parameters on Eurocodeapplied.com (link)

Weld throat thickness:
$a$

# STEEL DESIGN 

## ULS compression verification

```
Compressive design
resistance (EN 1993-1-1 (6.10)):
\[
\overline{N_{c . R d}}=\frac{A \cdot f_{y}}{\gamma_{M 0}}
\]
With,
```

A fulleffective cross-sectional area depending on the crosssection class
steel yield strength
partial factor
$\begin{aligned} & \text { Compression verification } \\ & \text { (EN 1993-1-1 (6.9): }\end{aligned} \quad \frac{N_{E d}}{N_{c . R d}} \leq 1.0$
With,
$N_{\text {Ed }}$
design compression force (normal force)

## ULS bending verification

Bending design resistance (EN 1993-1-1 (6.13)):

$$
M_{c . R d}=\frac{W_{p l} \cdot f_{y}}{\gamma_{M 0}}
$$

With, can find the value here.

Bending verification (EN 1993-1-1 (6.12):

$$
\frac{M_{E d}}{M_{C . R d}} \leq 1.0
$$

With,
$M_{E d}$ design bending moment

## STEEL DESIGN

## ULS shear verification

Shear design resistance (EN 1993-1-1 (6.18)):

$$
V_{p l . R d}=A_{v} \frac{\frac{f_{y}}{\sqrt{3}}}{\gamma_{M 0}}
$$

With,
$A_{v} \quad$ shear area. You can find the values here.

Check if shear buckling verification is required

$$
\varepsilon \leq \sqrt{\frac{235 M P a}{f_{y}}}
$$

(EN 1993-1-1 (6.22)):

$$
\eta \leq 1.0
$$

$$
\frac{h_{w}}{t_{w}} \leq 72 \cdot \frac{\varepsilon}{\eta}
$$

Shear verification
(EN 1993-1-1 (6.17):

$$
\frac{V_{E d}}{V_{c . R d}} \leq 1.0
$$

With,
$\mathrm{V}_{\mathrm{Ed}} \quad$ design shear force

## STEEL DESIGN

## ULS flexural buckling verification (columns)

Buckling length:

Radius of inertia:

Slenderness (EN 1993-1-1
6.3.1.3 (1)):
$l$
$i=\sqrt{\frac{I}{w \cdot h}}$
$\lambda_{1}=\pi \sqrt{\frac{E}{f_{y}}}$
Non-dimensional slenderness: $\quad \lambda=\frac{l}{i} \cdot \frac{1}{\lambda_{1}}$

Buckling curve (EN 1993-1-1 $\alpha$
Table 6.1):

Reduction factors

$$
\left.\phi=0.5 \cdot\left(1+\alpha \cdot(\lambda-0.2)+\lambda^{2}\right)\right)
$$

(EN 1993-1-1 (6.49)):

$$
\chi=\frac{1}{\phi+\sqrt{\phi^{2}-\lambda^{2}}}
$$

Design buckling resistance (EN 1993-1-1 (6.47))

$$
N_{b . R d}=\frac{\chi \cdot A \cdot f_{y}}{\gamma_{M 1}}
$$

With,

A full/effective cross-sectional area depending on the crosssection class

Utilization check (EN 1993-1-1 (6.46))

$$
\frac{N_{E d}}{N_{b . R d}} \leq 1
$$

## STEEL DESIGN

## Fillet weld design - directional method

Normal stress perpendicular $\quad \sigma_{90}=\frac{N}{2 \cdot l_{e f f} \cdot a \cdot \sqrt{2}}+\frac{M}{2 \cdot l_{e f f}^{2} \cdot \frac{a}{6} \cdot \sqrt{2}}$
to throat:

Shear stress parallel to the axis of weld:

$$
\tau_{0}=\frac{V}{2 \cdot l_{e f f} \cdot a}
$$

Shear stress perpendicular to

$$
\tau_{90}=\sigma_{90}
$$

the axis of weld:


Verification criteria 1 (EN 1993-1-8 (4.1)):

Verification criteria 2 (EN 1993-1-8 (4.1)):

$$
\sqrt{\sigma_{90}^{2}+3\left(\tau_{0}^{2}+\tau_{90}^{2}\right)} \leq \frac{f_{u}}{\beta_{w} \cdot \gamma_{M 2}}
$$

$$
\sigma_{90} \leq 0.9 \frac{f_{u}}{\gamma_{M 2}}
$$

## STEEL DESIGN

## Fillet weld design - simplified method

Normal stress in weld:

$$
\sigma_{N}=\frac{N}{l_{w} \cdot a}+\frac{M}{W}
$$

With,

| $I_{w}$ | length of weld |
| :--- | :--- |
| $W$ | section modulus of weld |

## Shear stress in weld:

Shear stress perpendicular to the axis of weld:

$$
\tau_{v}=\frac{V_{d}}{l_{w} \cdot a}
$$

Resulting stress:

$$
F_{w \cdot E d}=\sqrt{\sigma_{N}^{2}+\tau_{v}^{2}}
$$

Resistance stress:

$$
F_{w . R d}=\frac{f_{u}}{\sqrt{3} \cdot \beta_{w} \cdot \gamma_{M 2}}
$$

Utilization check

$$
F_{w \cdot E d} \leq F_{w . R d}
$$

(EN 1993-1-8 (4.2)):
Full in-depth article about fillet weld design here.

# RC DESIGN 

## Material properties

E-modulus concrete

E

(dependent on concrete
strength class):

Partial factor - concrete: $\quad \gamma_{c}$

Partial factor - reinforcement: $\gamma_{s}$

Characteristic cylinder
$f_{c k}$ compressive strength:
Design compressive strength: $f_{c d}=\frac{f_{c k}}{\gamma_{c}}$

Mean tensile concrete
$f_{c t m}$
strength:
Design tensile strength: $\quad f_{c t d}=\frac{f_{c t m}}{\gamma_{c}}$

Yield strength of reinforcement:
Design yield strength: $\quad f_{y d}=\frac{f_{y k}}{\gamma_{s}}$

You can find the values of these properties on Eurocodeapplied.com (link)

## RC DESIGN

## ULS Bending verification

Lever arm of longitudinal $d$
reinforcement to compression
fibre

$$
\begin{aligned}
& \mu=\frac{M_{d}}{w \cdot d^{2} \cdot \eta \cdot f_{c d}} \\
& \omega=1-\sqrt{1-2 \cdot \mu} \\
& A_{s . r e q}=\omega \cdot \frac{w \cdot d \cdot \eta \cdot f_{c d}}{f_{y d}}
\end{aligned}
$$

Required longitudinal reinforcement:

With,
$M_{d}$
w
$\eta$
bending moment
width of RC beam
factor dependent on concrete class

$$
\omega_{\min }=\max \left(0.26 \frac{f_{c t m}}{f_{y k}} \cdot \frac{f_{y d}}{f_{c d}} ; 0.0013 \frac{f_{y d}}{f_{c d}}\right)
$$

$$
\omega_{b a l}=\lambda \frac{\varepsilon_{c u 3}}{\varepsilon_{c u 3} \cdot \varepsilon_{y d}}
$$

$$
\omega_{\max }=0.044 \frac{f_{y d}}{\eta \cdot f_{c d}}
$$

Check 1:

$$
\omega>\omega_{\min }
$$

Check 2:
$\omega<\omega_{\text {bal }}$

## RC DESIGN

Check 3:

$$
\omega<\omega_{\max }
$$

Minimum reinforcement (EN 1992-1-1 9.2.1.1 (9.1N)):

$$
A_{s . \min }=\max \left\{\begin{array}{c}
0.26 \frac{f_{c t m}}{f_{y k}} \cdot w \cdot d \\
0.0013 \cdot w \cdot d
\end{array}\right.
$$

## ULS Shear verification

First, check if shear reinforcement is required.
Members not requiring design shear reinforcement
EN 1992-1-1 6.2.2 (1):

$$
\begin{aligned}
& k=1+\sqrt{\frac{200}{\frac{d}{m m}}} \\
& \rho_{1}=\min \left(\frac{A_{s}}{w \cdot d} ; 0.02\right) \\
& v_{\text {Rd.c }}=\max \left\{\begin{array}{c}
\frac{0.18}{\gamma_{c}} \cdot k \cdot\left(100 \rho_{1} \cdot f_{c k}\right)^{\frac{1}{3}} \\
\frac{0.051}{\gamma_{c}} \cdot k^{\frac{3}{2}} \cdot \sqrt{f_{c k}}
\end{array}\right. \\
& V_{\text {Rd.c }}=v_{\text {Rd.c }} \cdot w \cdot h
\end{aligned}
$$

Shear reinforcement required $\quad V_{E d}>V_{R d . c}$ if:

With, width of RC beam

## RC DESIGN

## Members requiring design shear reinforcement

EN 1992-1-1 Figure 6.5

$$
z=0.9 \cdot d
$$

Coefficient taking into
$\alpha_{c w}$
account state of stress in compression chord:

Strength reduction factor for $\quad v_{1}$ concrete cracked in shear (EN 1992-1-1 (6.9)):

Angle between concrete $\theta$
compression strut and beam
axis perp. to shear force:
Shear resistance (EN 1992-1-1 $\quad V_{R d . m a x}=\alpha_{c w} \cdot w \cdot z \cdot v_{1} \cdot \frac{f_{c d}}{\cot (\theta)+\tan (\theta)}$
(6.9)):

Verification
$\eta=\frac{V_{E d}}{V_{R d . \max }}$

Reduction of design yield strength of reinforcement (EN 1992-1-1 (6.8)):
Shear links (EN 1992-1-1 (6.8)): $\quad A_{s w}=\frac{V_{E d}}{z \cdot f_{y w d} \cdot \cot (\theta)}$

Inclination of shear $\alpha$ reinforcement:

Max. spacing (EN 1992-1-1 (9.6N)):

$$
f_{y w d}=0.8 \cdot f_{y k}
$$

$$
s_{l . \max }=0.75 \cdot d \cdot(1+\cot (\alpha))
$$

## RC DESIGN

## SLS Crack width verification

Crack width limit (EN 1992-1-1 $w_{\max }$
Table 7.1N)

Final creep coefficient $\phi$

E-modulus of concrete longterm (quasi-permanent):

$$
E_{c . e f f}=\frac{E_{c m}}{1+\phi}
$$

Long-term steel-concrete ratio:

$$
\alpha_{s}=\frac{E_{s}}{E_{c . e f f}}
$$



Equilibrium of 1. moment of area:
Neutral axis (solving for x ):

$$
\begin{aligned}
& w \cdot x \cdot \frac{x}{2}=\alpha_{s} \cdot A_{s} \cdot(d-x) \\
& x=\frac{\alpha_{s} \cdot A_{s}}{w} \cdot\left(-1+\sqrt{1+\frac{2 \cdot w \cdot d}{\alpha_{s} \cdot A_{s}}}\right)
\end{aligned}
$$

## RC DESIGN

Stress in reinforcement:

$$
\sigma_{s}=\frac{M_{q p}}{\left(d-\frac{x}{3}\right) \cdot A_{s}}
$$

Factor dependent on duration $k_{t}$ of load (EN 1992-1-1)

$$
f_{c t . e f f}=f_{c t m}
$$

Effective height $\left(E N 1992-1-1 \quad h_{c . e f f}=\min \left(2.5 \cdot(h-d) ; \frac{h-x}{3} ; \frac{h}{2}\right)\right.$
(7.3.2 (3)):

$$
\rho_{p . e f f}=\frac{A_{s}}{h_{c . e f f} \cdot w}
$$

EN 1992-1-1 (7.9):

$$
\varepsilon_{s m}-\varepsilon_{c m}=\max \left\{\begin{array}{c}
\frac{\sigma_{s}-k_{t} \cdot \frac{f_{c t . e f f}}{\rho_{\text {p.eff }}} \cdot\left(1+\alpha_{s} \cdot \rho_{\text {p.eff }}\right)}{E_{s}} \\
0.6 \cdot \frac{\sigma_{s}}{E_{s}}
\end{array}\right.
$$

Coefficient (EN 1992-1-1 (7.11)) $k_{1}$

Coefficient (EN 1992-1-1 (7.11)) $k_{2}$

Max. cracking spacing:

$$
s_{r . \max }=3.4 \cdot c+0.425 \cdot k_{1} \cdot k_{2} \cdot \frac{d_{s}}{\rho_{\text {p.eff }}}
$$

Verification:

$$
s_{r . \max }<5 \cdot\left(c+\frac{d_{s}}{2}\right)
$$

# RC DESIGN 

Crack width:

$$
w_{k}=s_{r . \max } \cdot\left(\varepsilon_{s m}-\varepsilon_{c m}\right)
$$

Utilization:

$$
\eta=\frac{w_{k}}{w_{\max }}
$$

## SLS Deflection verification

The deflection calculation of reinforced concrete elements is not as straightforward as for timber or steel structures. However, according to Eurocode EN 1992-1-1 the deflection requirement is likely to be satisfied if the span - effective depth ratio is less than the values given in EN 1992-1-1 Table 7.4N.

Click here for a reinforced concrete beam design guide.

