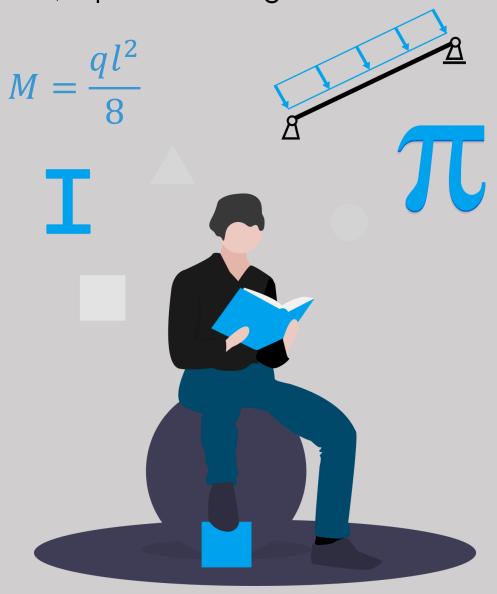


STRUCTURAL DESIGN CHEATSHEET

A summary of the most important formulas, equations & diagrams



DISCLAIMER

This Structural Design Cheatsheet covers the engineering formulas and equations I use regularly as a structural engineer. It's a "short" summary of many of the blog posts we published on structuralbasics.com.

While this document covers many important formulas, be aware that it doesn't cover every formula. And as for any structural engineering book or formula sheet, some explanations and formulas are simplified. For example, the wind load is only shown from one direction, or the steel Eurocode provides also formulas for bending and shear, normal force and bending, etc.

Structural engineering is simply too complex to cover every design situation.

Use the content of the document as a guide, but question everything critically.

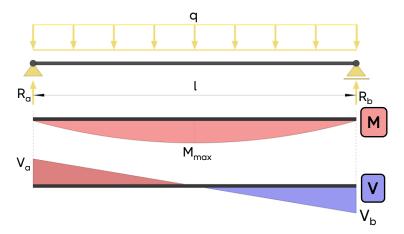
This document is supplied 'as-is,' without any express or implied warranty regarding the accuracy or completeness of the information it contains. The user assumes all risks associated with its use. The author will not be held responsible for any damage or harm to any person or entity that may come from utilizing this document.

With that out of the way, I hope this document helps you get a good overview of structural design.



STATICS

<u>Simply supported beam – Line load</u>



Reation forces:

$$R_a = R_b = q \frac{l}{2}$$

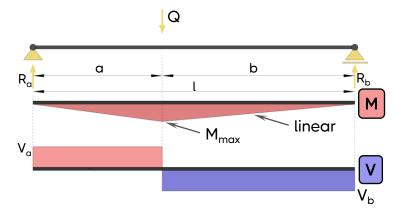
Max. shear forces:

$$V_a = V_b = q \frac{l}{2}$$

Max. bending moment:

$$M_{max} = q \frac{l^2}{8}$$

<u>Simply supported beam – Point load</u>



Reation forces:

$$R_a = Q \frac{b}{l}, R_b = Q \frac{a}{l}$$

Shear forces:

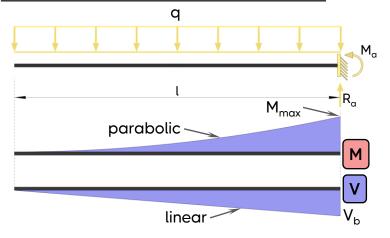
$$V_a = Q \frac{b}{l}, V_b = Q \frac{a}{l}$$

Max. bending moment:

$$M_{max} = Q \frac{a \cdot b}{l}$$

For more moment and shear force formulas of the simply supported beam click here.

<u>Cantilever beam – Line load</u>



Reation forces:

$$R_a = q \cdot l, M_a = -\frac{1}{2} \cdot q \cdot l^2$$

Shear forces:

$$V_a = -q \cdot l$$

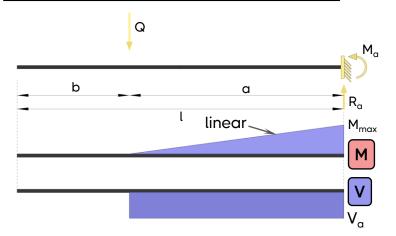
Max. bending moment:

$$M_{max} = -\frac{1}{2} \cdot q \cdot l^2$$



STATICS

Cantilever beam - Point load



Reation forces:

$$R_a = Q$$

Max. shear forces:

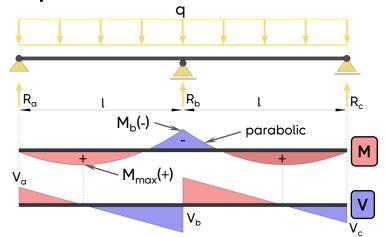
$$V_a = -Q$$

Max. bending moment:

$$M_{max} = Q \cdot a$$

For more moment and shear force formulas of the cantilever beam click here.

<u> 2 span continuous beam – Line load on 2 spans</u>



Reation forces:

$$R_a = R_c = \frac{3}{8}q \cdot l, R_b = \frac{5}{4}q \cdot l$$

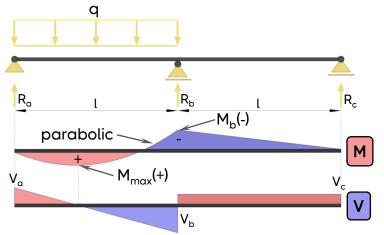
Shear forces

$$V_a = V_c = \frac{3}{8}q \cdot l, V_b = \frac{5}{8}q \cdot l$$

Bending moments:

$$M_{max} = \frac{9}{128} \cdot q \cdot l^2$$
, $M_b = -\frac{1}{8} \cdot q \cdot l^2$

2 span continuous beam – Line load on 1 span



Reation forces:

$$R_{\alpha} = \frac{7}{16}q \cdot l, R_{b} = \frac{5}{4}q \cdot l,$$

$$R_{c} = -\frac{1}{16}q \cdot l$$

Shear forces:

$$V_a = R_a, V_b = -\frac{9}{16}q \cdot l, V_c = -R_c$$

Max. bending moment:

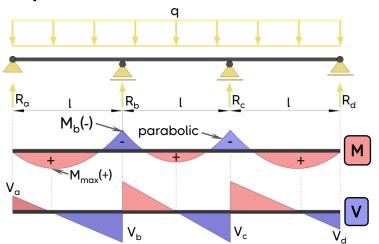
$$M_{max} = \frac{49}{512} \cdot q \cdot l^2, M_b = -\frac{1}{16} \cdot q \cdot l^2$$

For more moment and shear force formulas of the 2 span continuous beam click here.



STATICS

3 span continuous beam – Line load on 3 spans



Reation forces:

$$R_a = R_d = 0.4 \cdot q \cdot l,$$

$$R_b = R_c = 1.1 \cdot q \cdot l$$

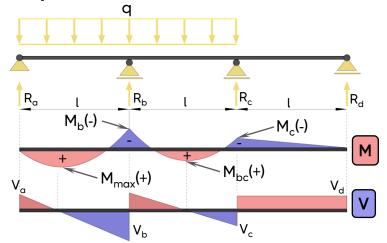
Shear forces:

$$\begin{split} V_{a} &= V_{d} = \pm 0.4 \cdot q \cdot l \,, \\ V_{b(-)} &= -V_{c(+)} = -0.6 \cdot q \cdot l \,, \\ V_{b(+)} &= -V_{c(-)} = 0.5 \cdot q \cdot l \end{split}$$

Bending moment:

$$M_{max} = 0.08 \cdot q \cdot l^2, M_b = -0.1 \cdot q \cdot l^2$$

3 span continuous beam – Line load on 2 spans



Reation forces:

$$R_a = 0.383 \cdot q \cdot l, R_b = 1.2 \cdot q \cdot l,$$

$$R_c = 0.45 \cdot q \cdot l, R_d = -0.033 \cdot q \cdot l$$

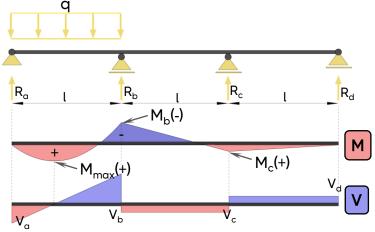
Shear forces:

$$\begin{array}{l} V_a = 0.383 \cdot q \cdot l \; , V_d = 0.033 \cdot q \cdot l \\ V_{b(-)} = -0.617ql \; , V_{b(+)} = -0.583ql \\ V_{c(-)} = -0.417ql \; , V_{c(+)} = 0.033ql \end{array}$$

Bending moments:

$$\begin{split} M_{max} &= 0.074ql^2, M_b = -0.117ql^2, \\ M_{bc} &= 0.053ql^2, M_c = -0.033ql^2 \end{split}$$

3 span continuous beam – Line load on 1 span



Reation forces:

$$R_a = 0.433 \cdot q \cdot l, R_b = 0.65 \cdot q \cdot l,$$

$$R_c = -0.1 \cdot q \cdot l, R_d = 0.017 \cdot q \cdot l$$

Bending moments:

$$M_{max} = 0.094ql^2, M_b = -0.067ql^2, M_c = 0.017ql^2$$

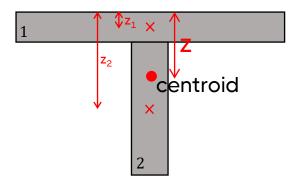
For more moment and shear force formulas of the 3 span continuous beam click here.



GEOMETRY

Centroid

The centroid is a point of a cross-section that represents the *center of mass*. It's the point at which the entire area of the section can be assumed to be concentrated.



$$z = \frac{\sum_{i=1}^{n} A_i \cdot z_i}{A}$$

With.

n number of "parts" of the cross-section (n=2 in the shape above)

A_i area of the part i

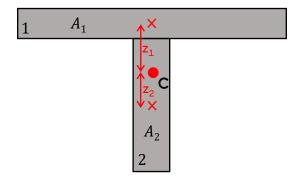
z_i distance of the centroid of part i from the point of origin (top fibre in

the shape above)

A area of the section

Moment of inertia

The moment of inertia is a measure of an object's resistance to changes in rotational motion. It is used to calculate the bending stresses that a structural element will experience when subjected to a load.



$$I = \sum_{i=1}^{n} I_{0.i} + A_i \cdot z_i^2$$

With,

number of "parts" of the cross-section (n=2 in the shape above)

I_{0 i} moment of inertia of part i

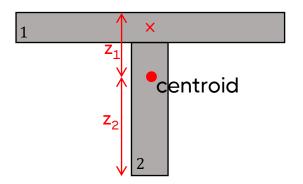
area of the part i

z_i distance of the centroid of part i from the centroid c



GEOMETRY

Section modulus



Section modulus of top fibre:

$$W = \frac{I}{z_1}$$

Section modulus of bottom fibre:

$$W = \frac{I}{z_2}$$

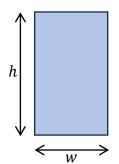
With,

I

moment of inertia around strong axis distance of fibre from centroid

Moment of inertia formulas

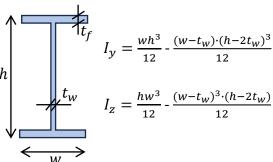
Rectangular section



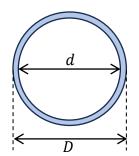
$$I_{y} = \frac{wh^3}{12}$$

$$I_z = \frac{hw^3}{12}$$

I/H section



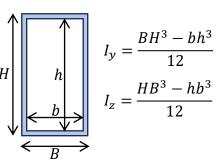
Hollow circular section



$$I_y = \frac{(D^4 - d^4) \cdot \pi}{64}$$

$$I_z = \frac{(D^4 - d^4) \cdot \pi}{64}$$

Rectangular hollow section



For more moment of inertia formulas of other sections click here.



Overview of loads used in structural design

The "common" characteristic loads that are used in the structural design of buildings are:

- Dead load
- Live load
- · Horizontal wind load on walls
- Wind loads on roofs
- Snow load
- Soil pressure
- Seismic load

Dead load

The dead load represents permanent loads, such as the self-weight of structural and non-structural building materials. The self-weight of a concrete slab, a timber truss roof and windows are examples of the dead load. The weight is calculated and then applied to the structural member that carries it.

Area dead load:

$$g_k = density \ of \ element \cdot thickness \ [\frac{kN}{m}]$$

Click <u>here</u> for the in-depth article.

Live load

The live load represents variable loads such as weight of people, furniture, cars, office equipment, etc. that can change over time. It's an approximation for structural engineers to estimate the additional weight (excluding self-weight) that can act on structures due to different room categories.

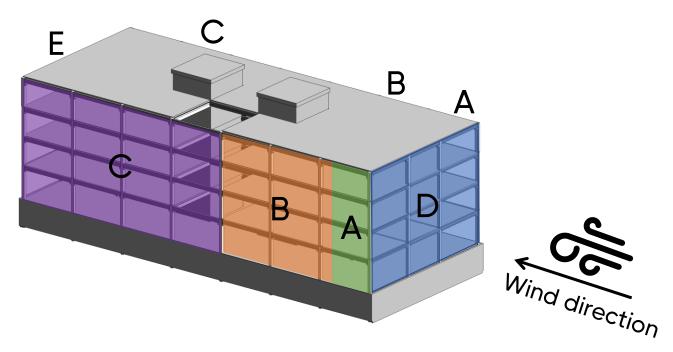
You'll find the values of the live load for different room classes in EN 1991-1-1 Table 6.2 and the National Annex of your country.

Click <u>here</u> for the in-depth article.



Horizontal wind loads on walls

The horizontal wind load on buildings are split up in 4 or 5 areas (A, B, C, D, E) with different load values.



Wind loads on walls:

$$w_k = q_p \cdot c_{pe}$$

With,

q_p peak velocity pressure

c_{pe} pressure coefficient for each area according to EN 1991-

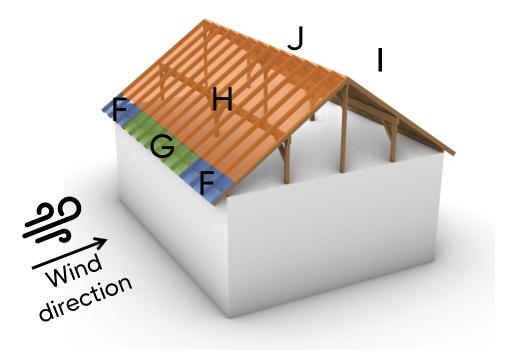
1-4 Table 7.1

Click <u>here</u> for step-by-step guide for the peak velocity pressure. And <u>here</u> for the full calculation guide for horizontal wind loads.



Wind loads on pitched roofs

The wind loads on pitched roofs are split up in 4 or 5 areas (F, G, H, I, (J)) with different load values.



Wind loads on pitched roofs:

$$w_k = q_p \cdot c_{pe}$$

With,

q_p peak velocity pressure

c_{pe} pressure coefficient for each area according to EN 1991-

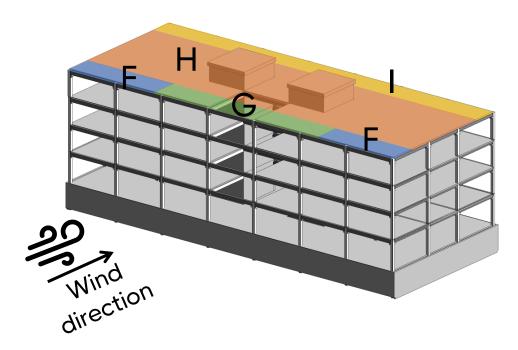
1-4 Table 7.4a.

Click here for the calculation guide for wind loads on pitched roofs.



Wind loads on flat roofs

The wind loads on flat roofs are split up in 4 areas (F, G, H, I) with different load values.



Wind loads on flat roofs:

$$w_k = q_p \cdot c_{pe}$$

With,

q_p peak velocity pressure

c_{pe} pressure coefficient for each area according to EN 1991-

1-4 Table 7.2.

Click <u>here</u> for the calculation guide for wind loads on flat roofs.



Snow loads on flat and pitched roofs

The snow load on pitched and flat roofs is calculated as (EN 1991-1-3 (5.1)):

$$s = \mu_i \cdot C_e \cdot C_t \cdot s_k$$

With,

 μ_{i} snow load shape coefficient

C_e exposure coefficient C_t thermal coefficient

s_k characteristic snow load value on ground

Click <u>here</u> for a calculation guide for snow loads on pitched roofs. And <u>here</u> for flat roofs.



LOAD COMBINATIONS

<u>Ultimate limit state (ULS)</u>

$$\sum_{j\geq 1} \gamma_{G,j} \cdot G_{k,j} + \gamma_{Q,1} \cdot Q_{k,1} + \sum_{i>1} \gamma_{Q,i} \cdot \psi_{0,i} \cdot Q_{k,i}$$

With,

 γ_{G} partial factor for permanent actions

G_k characteristic value of permanent action (e.g. dead load)

 γ_Q partial factor for variable actions

Q_k characteristic value of variable action (e.g. live, snow and

wind load)

 Ψ_0 factor for combination value of a variable action

Serviceability limit state (SLS)

Characteristic combination:

$$\sum_{j \ge 1} G_{k,j} + Q_{k,1} + \sum_{i > 1} \psi_{0,i} \cdot Q_{k,i}$$

Frequent combination:

$$\sum_{j\geq 1} G_{k,j} + \psi_{1,1} \cdot Q_{k,1} + \sum_{i>1} \psi_{2,i} \cdot Q_{k,i}$$

Quasi-permanent combination:

$$\sum_{j\geq 1} G_{k,j} + \sum_{i\geq 1} \psi_{2,i} \cdot Q_{k,i}$$

With,

 Ψ_1 factor for frequent value of a variable action

 Ψ_2 factor for quasi-permanent value of a variable action



LOAD COMBINATIONS

Accidental limit state (ALS)

$$\sum_{j\geq 1} G_{k,j} + A_d + (\psi_{1,1} \text{ or } \psi_{2,1}) \cdot Q_{k,1} + \sum_{i>1} \psi_{2,i} \cdot Q_{k,i}$$

Seismic combination:

$$\sum_{j\geq 1} G_{k,j} + A_{Ed} + \sum_{i\geq 1} \psi_{2,i} \cdot Q_{k,i}$$

With,

A_d design value of an accidental action

A_{Ed} design value of seismic action

Full in-depth article about load combinations: here



Bending verification

Design bending stresses:

$$\sigma_{m.y.d} = \frac{M_{y.d}}{I_y} \cdot \frac{h}{2}$$

$$\sigma_{m.z.d} = \frac{M_{z.d}}{I_z} \cdot \frac{w}{2}$$

With.

M_d design bending moment around y/z-axis

moment of inertia around y/z-axis

h cross-sectional height w cross-sectional width

Design bending strengths (EN 1995-1-1 (2.17)):

$$f_{m.y.d} = k_{mod} \frac{f_{m.y.k}}{\gamma_m}$$

$$f_{m.z.d} = k_{mod} \frac{f_{m.z.k}}{\gamma_m}$$

With,

k_{mod} modification factor (EN 1995-1-1 Table 3.1)

f_{m,k} characteristic bending strength of timber material

 γ_m partial factor (EN 1995-1-1 Table 2.3)

Bending verification (EN 1995-1-1 (6.11) + (6.12))

$$k_m = 0.7/1.0$$

EN 1995-1-1 6.1.6 (2)

$$\frac{\sigma_{m.y.d}}{f_{m.y.d}} + k_m \cdot \frac{\sigma_{m.z.d}}{f_{m.z.d}} \le 1$$

$$k_m \cdot \frac{\sigma_{m.y.d}}{f_{m.y.d}} + \frac{\sigma_{m.z.d}}{f_{m.z.d}} \le 1$$



Shear verification

Design shear stress:

$$\tau_{v.d} = \frac{3}{2} \cdot \frac{V_d}{w \cdot h}$$

With,

V_d design shear force h cross-sectional height w cross-sectional width

Design shear strengths (EN 1995-1-1 (2.17)):

$$f_{v.d} = k_{mod} \frac{f_{v.k}}{\gamma_m}$$

With,

k_{mod} modification factor (EN 1995-1-1 Table 3.1) f_{v,k} characteristic shear strength of timber material

γ_m partial factor (EN 1995-1-1 Table 2.3)

Shear verification (EN 1995-1-1 (6.13))

$$\frac{\tau_{v.d}}{f_{v.d}} \le 1$$



Compression verification (columns)

Design compression stress:

$$\sigma_{c.0.d} = \frac{N_d}{w \cdot h}$$

With,

N_d design normal force h cross-sectional height w cross-sectional width

Design compression strength – parallel to grain (EN 1995-1-1 (2.17)):

$$f_{c.0.d} = k_{mod} \frac{f_{c.0.k}}{\gamma_m}$$

With,

k_{mod} modification factor (EN 1995-1-1 Table 3.1)

f_{c.0.k} characteristic compression strength parallel to grain

 $\gamma_{\rm m}$ partial factor (EN 1995-1-1 Table 2.3)

Compression verification (EN 1995-1-1 (6.2))

$$\frac{\sigma_{c.0.d}}{f_{c.0.d}} \le 1$$



l

Buckling verification (columns)

Buckling length:

Radius of inertia: $i = \sqrt{\frac{I}{w \cdot h}}$

Slenderness ratio: $\lambda = \frac{l}{i}$

Relative slenderness ratio (EN $\lambda_{rel.y} = \frac{\lambda}{\pi} \cdot \sqrt{\frac{f_{c.0.k}}{E_{0.g.05}}}$

 β_c factor (EN 1995-1-1 (6.29)) β_c

Instability factor (EN 1995-1-1 $k = 0.5 \cdot (1 + \beta_c \cdot (\lambda_{rel.y} - 0.3) + \lambda_{rel.y}^2)$ (6.27))

Buckling reduction factor coefficient (EN 1995-1-1 (6.25)) $k_c = \frac{1}{k + \sqrt{k^2 - \lambda_{rel.y}^2}}$

Utilization check one axial bending (EN 1995-1-1 (6.23)) $\frac{\sigma_{c.0.d}}{k_c \cdot f_{c.0.d}} + \frac{\sigma_{m.d}}{f_{m.d}} \leq 1$

With,

 $f_{c.0.k}$ characteristic compression strength parallel to grain

E-modulus parallel to grain, 5% fractile

moment of inertia
h cross-sectional height
w cross-sectional width



Material properties

 $E = 0.21 \cdot 10^6 MPa$ E-modulus:

 $\rho = 7850 \frac{kg}{m^3}$ Density:

 $\nu = 0.3$ Poissons ratio:

 $G = \frac{E}{2 \cdot (1 + \nu)}$ Shear modulus:

 f_{y} , f_{u} Yield and ultimate strength:

Check out the tables on Eurocodeapplied.com (link)

Strength properties of bolts (EN 1993-1-8 Table 3.1)

	Bolt class						
Property	4.6	4.8	5.6	5.8	6.8	8.8	10.9
Yield strength f _{yb} (MPa)	240	320	300	400	480	640	900
Ultimate tensile strength f _{ub} (MPa)	400	400	500	500	600	800	1000

Ultimate tensile strength of $f_u = f_u$ of the weaker steel plate

welds:

Correlation factor (dependent on steel strength (EN 1993-1-8 Table 4.1)

 β_0



Geometric properties Cross-sectional height: h w Cross-sectional width: Web thickness: t_{w} Flange thickness: t_f Root radius: r You can find these and many more parameters on Eurocodeapplied.com (link) d Bolt nominal diameter: Bolt Hole diameter: d_0 Stress area (threaded part of bolt): You can find these and many more parameters on Eurocodeapplied.com (link) Weld throat thickness: \boldsymbol{a}



ULS compression verification

Compressive design $N_{c.Rd} = \frac{A \cdot f_y}{\gamma_{M0}}$ resistance (EN 1993-1-1 (6.10)):

With,

A full/effective cross-sectional area depending on the cross-

section class

f_v steel yield strength

γ_{M0} partial factor

Compression verification

(EN 1993-1-1 (6.9):

 $\frac{N_{Ed}}{N_{c,Rd}} \le 1.0$

With,

N_{Ed} design compression force (normal force)

ULS bending verification

Bending design resistance (EN 1993-1-1 (6.13)):

$$M_{c.Rd} = \frac{W_{pl} \cdot f_{y}}{\gamma_{M0}}$$

With,

W_{pl} plastic section modulus (cross-section classes 1+2). You

can find the value here.

Bending verification (EN 1993-1-1 (6.12):

$$\frac{M_{Ed}}{M_{c.Rd}} \le 1.0$$

With,

M_{Ed} design bending moment



ULS shear verification

Shear design resistance (EN 1993-1-1 (6.18)):

$$V_{pl.Rd} = A_v \frac{\frac{f_y}{\sqrt{3}}}{\gamma_{M0}}$$

With,

 A_{v}

shear area. You can find the values here.

Check if shear buckling verification is required (EN 1993-1-1 (6.22)):

$$\varepsilon \leq \sqrt{\frac{235MPa}{f_y}}$$

$$\eta \leq 1.0$$

$$\frac{h_w}{t_w} \le 72 \cdot \frac{\varepsilon}{\eta}$$

Shear verification (EN 1993-1-1 (6.17):

$$\frac{V_{Ed}}{V_{c,Rd}} \le 1.0$$

With,

 V_{Ed}

design shear force



ULS flexural buckling verification (columns)

Buckling length:

l

Radius of inertia:

$$i = \sqrt{\frac{I}{w \cdot h}}$$

Slenderness (EN 1993-1-1 6.3.1.3 (1)):

$$\lambda_1 = \pi \sqrt{\frac{E}{f_y}}$$

O.3.1.3 (1)):

$$\lambda = \frac{l}{i} \cdot \frac{1}{\lambda_1}$$

Non-dimensional slenderness:

Buckling curve (EN 1993-1-1 Table 6.1):

α

Reduction factors

(EN 1993-1-1 (6.49)):

$$\phi = 0.5 \cdot (1 + \alpha \cdot (\lambda - 0.2) + \lambda^2))$$

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \lambda^2}}$$

Design buckling resistance (EN 1993-1-1 (6.47))

$$N_{b.Rd} = \frac{\chi \cdot A \cdot f_y}{\gamma_{M1}}$$

With,

Α

full/effective cross-sectional area depending on the crosssection class

Utilization check (EN 1993-1-1 (6.46))

$$\frac{N_{Ed}}{N_{h,Rd}} \le 1$$



Fillet weld design - directional method

Normal stress perpendicular to throat:

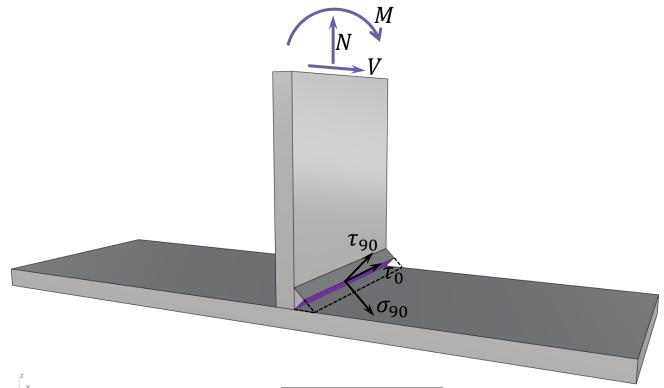
$$\sigma_{90} = \frac{N}{2 \cdot l_{eff} \cdot a \cdot \sqrt{2}} + \frac{M}{2 \cdot l_{eff}^2 \cdot \frac{a}{6} \cdot \sqrt{2}}$$

Shear stress parallel to the $au_0 = rac{V}{2 \cdot l_{eff} \cdot a}$ axis of weld:

$$\tau_0 = \frac{V}{2 \cdot l_{eff} \cdot a}$$

Shear stress perpendicular to $au_{90} = \sigma_{90}$ the axis of weld:

$$\tau_{90} = \sigma_{90}$$



Verification criteria 1 (EN 1993-1-8 (4.1)):

$$\sqrt{\sigma_{90}^2 + 3(\tau_0^2 + \tau_{90}^2)} \le \frac{f_u}{\beta_w \cdot \gamma_{M2}}$$

Verification criteria 2 (EN 1993-1-8 (4.1)):

$$\sigma_{90} \le 0.9 \frac{f_u}{\gamma_{M2}}$$



Fillet weld design - simplified method

Normal stress in weld:

$$\sigma_N = \frac{N}{l_w \cdot a} + \frac{M}{W}$$

With.

W

l_w,

length of weld

section modulus of weld

Shear stress in weld:

Shear stress perpendicular to $au_v = rac{V_d}{l_w \cdot a}$ the axis of weld:

$$\tau_v = \frac{V_d}{l_w \cdot a}$$

Resulting stress:

$$F_{w.Ed} = \sqrt{\sigma_N^2 + \tau_v^2}$$

Resistance stress:

$$F_{w.Rd} = \frac{f_u}{\sqrt{3} \cdot \beta_w \cdot \gamma_{M2}}$$

Utilization check (EN 1993-1-8 (4.2)):

$$F_{w.Ed} \le F_{w.Rd}$$

Full in-depth article about fillet weld design here.



Material properties

E-modulus concrete E (dependent on concrete

strength class):

Partial factor – concrete: γ_c

Partial factor – reinforcement: γ_s

Characteristic cylinder f_{ch}

compressive strength:

Design compressive strength: $f_{cd} = \frac{f_{ck}}{\gamma_c}$

Mean tensile concrete f_{ctm}

strength:

Design tensile strength: $f_{ctd} = \frac{f_{ctm}}{\gamma_c}$

Yield strength of f_{yk}

reinforcement:

Design yield strength: $f_{yd} = \frac{f_{yk}}{\gamma_s}$

You can find the values of these properties on Eurocodeapplied.com (<u>link</u>)



ULS Bending verification

Lever arm of longitudinal reinforcement to compression fibre

d

$$\mu = \frac{M_d}{w \cdot d^2 \cdot \eta \cdot f_{cd}}$$

$$\omega = 1 - \sqrt{1 - 2 \cdot \mu}$$

Required longitudinal reinforcement:

$$A_{s.req} = \omega \cdot \frac{w \cdot d \cdot \eta \cdot f_{cd}}{f_{yd}}$$

With,

M_d bending moment w width of RC beam

 η factor dependent on concrete class

Degree of reinforcement checks:

$$\omega_{min} = \max(0.26 \frac{f_{ctm}}{f_{yk}} \cdot \frac{f_{yd}}{f_{cd}}; 0.0013 \frac{f_{yd}}{f_{cd}})$$

$$\omega_{bal} = \lambda \frac{\varepsilon_{cu3}}{\varepsilon_{cu3} \cdot \varepsilon_{yd}}$$

$$\omega_{max} = 0.044 \frac{f_{yd}}{\eta \cdot f_{cd}}$$

Check 1: $\omega > \omega_{min}$

Check 2: $\omega < \omega_{bal}$

Structural Basics

Check 3:

 $\omega < \omega_{max}$

Minimum reinforcement (EN 1992-1-1 9.2.1.1 (9.1N)):

$$A_{s.min} = max \begin{cases} 0.26 \frac{f_{ctm}}{f_{yk}} \cdot w \cdot d \\ 0.0013 \cdot w \cdot d \end{cases}$$

ULS Shear verification

First, check if shear reinforcement is required.

Members not requiring design shear reinforcement

EN 1992-1-1 6.2.2 (1):

$$k = 1 + \sqrt{\frac{200}{\frac{d}{mm}}}$$

Design value of shear resistance (EN 1992-1-1 (6.2.a):

$$\rho_{1} = \min(\frac{A_{s}}{w \cdot d}; 0.02)$$

$$v_{Rd.c} = max \begin{cases} \frac{0.18}{\gamma_{c}} \cdot k \cdot (100\rho_{1} \cdot f_{ck})^{\frac{1}{3}} \\ \frac{0.051}{\gamma_{c}} \cdot k^{\frac{3}{2}} \cdot \sqrt{f_{ck}} \end{cases}$$

$$V_{Rd.c} = v_{Rd.c} \cdot w \cdot h$$

Shear reinforcement required $V_{Ed} > V_{Rd,c}$ if:

$$V_{Ed} > V_{Rd.c}$$

With.

h V_{Ed}

height of RC beam width of RC beam



Members requiring design shear reinforcement

EN 1992-1-1 Figure 6.5

$$z = 0.9 \cdot d$$

Coefficient taking into account state of stress in compression chord:

$$\alpha_{cw}$$

Strength reduction factor for concrete cracked in shear (EN 1992-1-1 (6.9)):

$$v_1$$

Angle between concrete compression strut and beam axis perp. to shear force:

$$\theta$$

Shear resistance (EN 1992-1-1 (6.9)):

$$V_{Rd.max} = \alpha_{cw} \cdot w \cdot z \cdot v_1 \cdot \frac{f_{cd}}{\cot(\theta) + \tan(\theta)}$$

Verification

$$\eta = \frac{V_{Ed}}{V_{Rd.max}}$$

Reduction of design yield strength of reinforcement (EN 1992-1-1 (6.8)):

$$f_{ywd} = 0.8 \cdot f_{yk}$$

Shear links (EN 1992-1-1 (6.8)):

$$A_{sw} = \frac{V_{Ed}}{z \cdot f_{ywd} \cdot \cot(\theta)}$$

Inclination of shear reinforcement:

Max. spacing (EN 1992-1-1 (9.6N)):

$$s_{l.max} = 0.75 \cdot d \cdot (1 + \cot(\alpha))$$



SLS Crack width verification

Crack width limit (EN 1992-1-1 w_{max} Table 7.1N)

Final creep coefficient

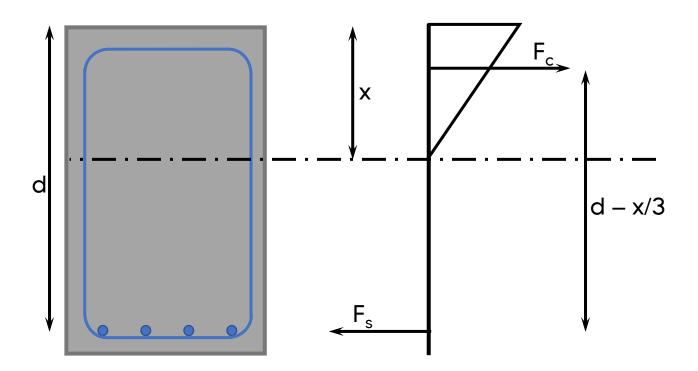
φ

E-modulus of concrete longterm (quasi-permanent):

$$E_{c.eff} = \frac{E_{cm}}{1 + \phi}$$

Long-term steel-concrete ratio:

$$\alpha_{s} = \frac{E_{s}}{E_{c.eff}}$$



Equilibrium of 1. moment of area:

 $w \cdot x \cdot \frac{x}{2} = \alpha_s \cdot A_s \cdot (d - x)$

Neutral axis (solving for x):

$$x = \frac{\alpha_s \cdot A_s}{w} \cdot (-1 + \sqrt{1 + \frac{2 \cdot w \cdot d}{\alpha_s \cdot A_s}})$$



Stress in reinforcement:

$$\sigma_s = \frac{M_{qp}}{\left(d - \frac{x}{3}\right) \cdot A_s}$$

Factor dependent on duration k_t of load (EN 1992-1-1)

$$f_{ct.eff} = f_{ctm}$$

(7.3.2(3)):

Effective height (EN 1992-1-1
$$h_{c.eff} = \min(2.5 \cdot (h-d); \frac{h-x}{3}; \frac{h}{2})$$

$$\rho_{p.eff} = \frac{A_S}{h_{c.eff} \cdot w}$$

EN 1992-1-1 (7.9):

$$\varepsilon_{sm} - \varepsilon_{cm} = \max \begin{cases} \frac{\sigma_s - k_t \cdot \frac{f_{ct.eff}}{\rho_{p.eff}} \cdot (1 + \alpha_s \cdot \rho_{p.eff})}{E_s} \\ 0.6 \cdot \frac{\sigma_s}{E_s} \end{cases}$$

Coefficient (EN 1992-1-1 (7.11)) k_1

Coefficient (EN 1992-1-1 (7.11)) k_2

Max. cracking spacing:

$$s_{r.max} = 3.4 \cdot c + 0.425 \cdot k_1 \cdot k_2 \cdot \frac{d_s}{\rho_{p.eff}}$$

Verification:

$$s_{r.max} < 5 \cdot (c + \frac{d_s}{2})$$



Crack width: $w_k = s_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm})$

Utilization: $\eta = \frac{w_k}{w_{max}}$

SLS Deflection verification

The deflection calculation of reinforced concrete elements is not as straightforward as for timber or steel structures. However, according to Eurocode EN 1992-1-1 the deflection requirement is likely to be satisfied if the span – effective depth ratio is less than the values given in EN 1992-1-1 Table 7.4N.

Click **here** for a reinforced concrete beam design guide.

